“Equations You Should Know”

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Test Concepts

• Basic Doppler hemodynamic calculations
• Doppler valve area calculations
• Quantification of regurgitant lesions
• Ventricular Function

• WATCH THE UNITS!!!
• (physics we leave to Sid)
Doppler Hemodynamic Calculations

• Volumetric flow
  – Stroke volume
  – Cardiac output/index
  – Regurgitant volumes and regurgitant fractions
  – Stenotic areas
  – Shunt fractions

Doppler Hemodynamic Calculations

• Basic Equation
  – $Q = V \times CSA$
    • $Q =$ volumetric flow (ml/s)
    • $V =$ velocity (cm/s)
    • $CSA =$ cross sectional area (cm$^2$)
    – Circular: $\pi R^2 = \pi (D/2)^2 = 0.785D^2$
Doppler Hemodynamic Calculations

- Ask you to calculate either SV or CO
  - Will be given VTI (cm), LVOT diameter (cm), HR along with other distracters
  - SV = VTI x CSA
  - CO = SV x HR
  - CI = (SV x HR)/BSA

Question

- Evaluation of a patient in the ICU reveals the following:
  - HR 80/min
  - LVOT diameter 2.0 cm
  - LVOT VTI 15 cm
  - Mitral VTI 12 cm
  - BP 130/70 mm Hg
- What is the cardiac output?
  - A. 3.0 L/min
  - B. 3.3 L/min
  - C. 3.8 L/min
  - D. 4.0 L/min
  - E. 4.5 l/min
Doppler Hemodynamic Calculations

• Continuity Principle
  – Conservation of mass
    • What mass flows in must flow out
  – Used in calculations of stenotic orifice areas and regurgitant orifice areas and regurgitant volumes and fractions

• BASIC EQUATION
  – $Q_1 = Q_2$
    • $CSA_1 \times VTI_1 = CSA_2 \times VTI_2$

Continuity Principle

$A_2 = \frac{A_1 \cdot v_1}{v_2}$

Question

• A 70 year-old man is being evaluated for aortic stenosis. The following measurements were obtained.
  – LVOT diameter 2.0 cm
  – AV peak gradient 48 mm Hg
  – Aortic valve mean gradient 32 mm Hg
  – Aortic valve VTI 63 cm
  – LVOT VTI 20 cm

• The AVA is calculated to be which of the following?
  – A. 0.6 cm²
  – B. 0.8 cm²
  – C. 1.0 cm²
  – D. 1.2 cm²
  – E. 1.4 cm²

Continuity Principle

• Proximal Isovelocity Surface Area (PISA)
  – Estimates volumetric flow based on color flow Doppler and spectral Doppler measurements
  – As flow approaches a restrictive orifice, isovelocity shells of increasing velocity and reducing area (radius) are formed.

• Used most commonly in calculations of regurgitant orifice area (EROA), regurgitant volumes, and regurgitant fractions.
  – Can be used to calculate stenotic orifice areas
PISA—Basic Equation

\[ Q = 2\pi r^2 \times V_a \]


Flow Convergence Method

\[ \text{Reg Flow} = 2\pi r^2 \times V_a \]
\[ \text{EROA} = \text{Reg Flow} / PkV_{\text{Reg}} \]
Bernoulli Equation

Generally can ignore flow acceleration and viscous friction terms

Modified Bernoulli Equation

\[ \Delta P = 4V^2 \]

If \( V_1 > 1.5 \text{ m/sec} \) it must be included!!

\[ \Delta P = 4(V_2^2 - V_1^2) \]

Valve Area Calculations—Stenotic Valve Lesions
Valve Area Calculations

- **Continuity Equation**
  \[ \text{CSA}_{\text{prox}} \times \text{VTI}_{\text{prox}} = \text{CSA}_{\text{sten}} \times \text{VTI}_{\text{sten}} \]
  
  \[ \text{CSA}_{\text{sten}} = \frac{(\text{CSA}_{\text{prox}} \times \text{VTI}_{\text{prox}})}{\text{VTI}_{\text{sten}}} \]

Aortic Valve Area

- **AVA** = \( \frac{(\text{CSA}_{\text{LVOT}} \times \text{VTI}_{\text{LVOT}})}{\text{VTI}_{\text{AV}}} \)
Dimensionless Velocity Ratio

DVR = 

Table 3: Recommendations for classification of AS severity

<table>
<thead>
<tr>
<th></th>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aortic jet velocity (m/s)</td>
<td>≤2.5 m/s</td>
<td>2.6-2.9</td>
<td>3.0-4.0</td>
</tr>
<tr>
<td>Mean gradient (mmHg)</td>
<td>≤20 (&lt;30°)</td>
<td>20-40° (30-50°)</td>
<td>&gt;40° (&gt;50°)</td>
</tr>
<tr>
<td>AVA (cm²)</td>
<td>&gt;1.5</td>
<td>1.0-1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Indexed AVA (cm²/m²)</td>
<td>&gt;0.85</td>
<td>0.60-0.85</td>
<td>&lt;0.6</td>
</tr>
<tr>
<td>Velocity ratio</td>
<td>&gt;0.50</td>
<td>0.25-0.50</td>
<td>&lt;0.25</td>
</tr>
</tbody>
</table>

*ESC Guidelines.
*AHA/ACC Guidelines.

Mitral Valve Area

- Pressure half-time
  - \( \text{MVA} = \frac{220}{\text{PHT}} \)
- Deceleration time
  - \( \text{PHT} = 0.29 \times \text{DT} \)
  - \( \text{MVA} = \frac{759}{\text{DT}} \)

**Question**

The best estimate of the mitral orifice area is:

- A. 0.40 cm\(^2\)
- B. 0.75 cm\(^2\)
- C. 1.0 cm\(^2\)
- D. 1.4 cm\(^2\)
- E. 2.6 cm\(^2\)
Mitral Valve Area

- Continuity Equation
  \[ \text{MVA} = \frac{(\text{CSA}_{\text{LVOT}} \times \text{VTI}_{\text{LVOT}})}{\text{VTI}_{\text{MV}}} \]

Mitral Valve Area—PISA

By continuity, boundary flow equals flow across the stenotic mitral orifice (MVA x Vp)
- Thus: \(2\pi R^2 \times V_n = \text{MVA} \times V_p\)
- Rearranging:
  - MVA = \(2\pi R^2 \times V_n / V_p\)
  - Due to doming, angle correction must be used

\[ \text{MVA} = \frac{(2\pi R^2 \times V_n)}{V_p \times (\alpha / 180)} \]
Table 9 Recommendations for classification of mitral stenosis severity

<table>
<thead>
<tr>
<th></th>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific findings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valve area (cm²)</td>
<td>&gt;1.5</td>
<td>1.0–1.5</td>
<td>&lt;1.0</td>
</tr>
<tr>
<td>Supportive findings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean gradient (mmHg) a</td>
<td>&lt;5</td>
<td>5–10</td>
<td>&gt;10</td>
</tr>
<tr>
<td>Pulmonary artery pressure (mmHg)</td>
<td>&lt;30</td>
<td>30–50</td>
<td>&gt;50</td>
</tr>
</tbody>
</table>

aAt heart rates between 60 and 80 bpm and in sinus rhythm.

Quantification of Regurgitant Lesions

Instantaneous flow rate = surface area $\times$ velocity = $A \times V$ ml/s
Stroke volume = $A \times VTI$ ml

Irvine T et al. Heart 2002;88(Suppl IV):iv11
Continuity

Irvine T et al. Heart 2002;88(Suppl IV):iv11

Mitral Regurgitation
Pulsed Doppler Quantitation

- Stroke volume = CSA x VTI
  - $\pi D^2/4 \times VTI = 0.785 D^2 \times VTI$
- Regurgitant volume (RVol)
  - $\text{RVol} = \text{SV}_{RV} - \text{SV}_{CV}$
  - $\text{RVol} = \text{MV inflow} - \text{AV outflow}$
  - $(\text{CSA}_{MV} \times \text{VTI}_{MV}) - (\text{CSA}_{LVOT} \times \text{VTI}_{LVOT})$
- Regurgitant fraction (RF)
  - $\text{RF} = (\text{RVol} / \text{MV flow}) \times 100$
- Effective Regurgitant Orifice Area (ERO)
  - $\text{ERO} = \text{RVol}/\text{VTI}_{MR}$
Proximal Isovelocity Surface Area (PISA) Method

By continuity, boundary flow 
\( (2\pi R^2 \times V_a) \) equals flow across the regurgitant orifice 
\( (ERO \times V_p) \)

\[
EROA = \frac{2\pi R^2 \times V_a}{V_p}
\]
Let’s Do the Math!

- \( ERO_{MR} = 2\pi R^2 \times (V_N / V_P) \)
  - \( ERO_{MR} = 6.28 \times 0.54 \times 0.54 \times (32/439) \)
  - \( ERO_{MR} = 0.13 \text{ cm}^2 \)

- **Simplified method**
  - Set Nyquist limit at 40 cm/s and assume that MR velocity is 500 cm/s
  - Then: \( ERO = R^2 / 2 \)

### Summary: MR Severity

<table>
<thead>
<tr>
<th>Specific signs of severity</th>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Small central jet &lt; 4 cm(^2)</td>
<td>Signs of MR &gt; mild present, but no criteria for severe MR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Vena contracta width &lt; 0.3 cm</td>
<td>Vena contracta width ≥ 0.7 cm with large central MR jet (area &gt; 40% of LA) or with a wall-impinging jet of any size, swirling in LA(^a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• No or minimal flow convergence(^b)</td>
<td>Large flow convergence(^b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Systolic reversal in pulmonary veins</td>
<td>Systolic reversal in pulmonary veins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Prominent flail MV leaflet or ruptured papillary muscle</td>
<td>Prominent flail MV leaflet or ruptured papillary muscle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supportive signs</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Systolic dominant flow in pulmonary veins</td>
<td>Intermediate signs/findings</td>
<td></td>
</tr>
<tr>
<td>• A-wave dominant mitral inflow(^c)</td>
<td>Dense, triangular CW</td>
<td></td>
</tr>
<tr>
<td>• Soft density, parabolic CW</td>
<td>Doppler MR jet</td>
<td></td>
</tr>
<tr>
<td>• Doppler MR signal</td>
<td>E-wave dominant mitral inflow (E &gt; 1.2 m/s)(^b)</td>
<td></td>
</tr>
<tr>
<td>• Normal LV size*</td>
<td>Enlarged LV and LA size**, particularly when normal LV function is present.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantitative parameters(^a)</th>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>R Vol (ml/bcat)</td>
<td>&lt; 30</td>
<td>30-44</td>
<td>45-59</td>
</tr>
<tr>
<td>RF (%)</td>
<td>&lt; 30</td>
<td>30-39</td>
<td>40-49</td>
</tr>
<tr>
<td>EROA (cm(^2))</td>
<td>&lt; 0.20</td>
<td>0.20-0.29</td>
<td>0.30-0.39</td>
</tr>
<tr>
<td></td>
<td>≥ 0.40</td>
<td>≥ 0.40</td>
<td></td>
</tr>
</tbody>
</table>
Aortic Regurgitation

– Quantitative Doppler
  • Regurgitant volume (RVol)
    – RVol = AV flow - MV flow
      » AV flow = LVOT annulus area x AV VTI
      » MV flow = MV annulus area x MV VTI
  • Regurgitant fraction (RF)
    – RF = (RVol / AV flow) x 100

Aortic Regurgitation: Quantification

• Proximal Isovelocity Surface Area method
  – Continuity principle
  – Effective regurgitant orifice (ERO) area
    • EROA = flow rate/peak AR velocity
    • $\text{EROA} = \left( \frac{2 \pi R^2 \times V_{\text{alias}}}{V_{\text{peak}}} \right)$
Let’s Do the Math!

- $\text{ERO}_{AR} = 2\pi R^2 \times \left( V_{\text{alias}} / V_{\text{peak}} \right)$
  - $\text{ERO}_{AR} = 6.28 \times (0.52 \times 0.52) \times (37/483)$
  - $\text{ERO}_{AR} = 0.13 \text{ cm}^2$
## Summary: AR Severity

<table>
<thead>
<tr>
<th></th>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific signs for AR severity</strong></td>
<td>Central Jet, width &lt; 25% of LVOT³</td>
<td>Signs of AR-&gt;mild present but no criteria for severe AR</td>
<td>Central Jet, width ≥ 65% of LVOT³</td>
</tr>
<tr>
<td></td>
<td>Vena contracta &lt; 0.3 cm³</td>
<td></td>
<td>Vena contracta &gt; 0.6cm³</td>
</tr>
<tr>
<td></td>
<td>No or brief early diastolic flow reversal in descending aorta</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Supportive signs</strong></td>
<td>Pressure half-time &gt; 500 ms</td>
<td>Intermediate values</td>
<td>Pressure half-time &lt; 200 ms</td>
</tr>
<tr>
<td></td>
<td>Normal LV size*</td>
<td></td>
<td>Holodiastolic aortic flow reversal in descending aorta</td>
</tr>
<tr>
<td><strong>Quantitative parameters</strong></td>
<td>R Vol, ml/beat</td>
<td>30-44</td>
<td>45-59</td>
</tr>
<tr>
<td>R Vol, ml/beat</td>
<td>&lt; 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RE, %</td>
<td>&lt; 30</td>
<td>30-39</td>
<td>40-49</td>
</tr>
<tr>
<td>EROA, cm²</td>
<td>&lt; 0.10</td>
<td>0.10-0.19</td>
<td>0.20-0.29</td>
</tr>
</tbody>
</table>

## Ventricular Function
### Specific signs of severity

<table>
<thead>
<tr>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small central jet &lt; 4 cm² or &lt; 20% of LA area*</td>
<td>Signs of MR=mild present, but no criteria for severe MR</td>
<td>Veuca, transa width ≥ 0.7 cm with large central MR jet (area &gt; 40% of LA) or with a wall-impinging jet of any size, swelling in LA*</td>
</tr>
<tr>
<td>Veuca transa width &lt;0.3 cm</td>
<td></td>
<td>Large flow convergence*</td>
</tr>
<tr>
<td>Ne or minimal flow convergence*</td>
<td></td>
<td>Systolic reversal in pulmonary veins</td>
</tr>
</tbody>
</table>

### Supportive signs

- Systolic dominant flow in pulmonary veins
- A-wave dominant mitral inflow*<sup>+</sup>
- Soft density, parabolic CW Doppler MR signal
- Normal LV size*<sup>+</sup>

### Quantitative parameters

<table>
<thead>
<tr>
<th>Mild</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWave (cm²)</td>
<td>&lt; 0.20</td>
<td>0.20-0.29</td>
</tr>
<tr>
<td>E/A Wave</td>
<td>&lt; 20</td>
<td>30-40</td>
</tr>
<tr>
<td>EF (%)</td>
<td>&gt; 40</td>
<td>40-40</td>
</tr>
</tbody>
</table>
How to do PISA-III

- Measure the peak velocity of the MR jet as parallel to flow as possible
- Trace the spectral profile to determine the VTI of the MR jet
Mitral Stenosis