“Equations You Should Know”

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Test Concepts

- Basic Doppler hemodynamic calculations
- Doppler valve area calculations
- Quantification of regurgitant lesions
- Ventricular Function

- **WATCH THE UNITS!!!**
- (physics we leave to Sid)

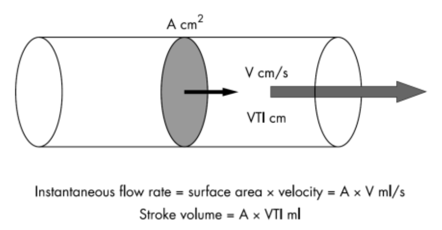
Doppler Hemodynamic Calcualtions

- Volumetric flow
  - Stroke volume
  - Cardiac output/index
  - Regurgitant volumes and regurgitant fractions
  - Stenotic areas
  - Shunt fractions

Doppler Hemodynamic Calculations

- Basic Equation
  - **$Q = V \times CSA$**
    - Q = volumetric flow (ml/s)
    - V = velocity (cm/s)
    - CSA = cross sectional area (cm^2)
      - Circular– $\pi R^2 = \pi(D/2)^2 = 0.785D^2$

Doppler Hemodynamic Calculations



Instantaneous flow rate = surface area × velocity = A × V ml/s
Stroke volume = A × VTI ml

Irvine T et al. Heart 2002;88(Suppl IV):iv11.

- Ask you to calculate either SV or CO
  - Will be given VTI (cm), LVOT diameter (cm), HR along with other distracters
  - SV = VTI x CSA
  - CO = SV x HR
  - CI = (SV x HR)/BSA

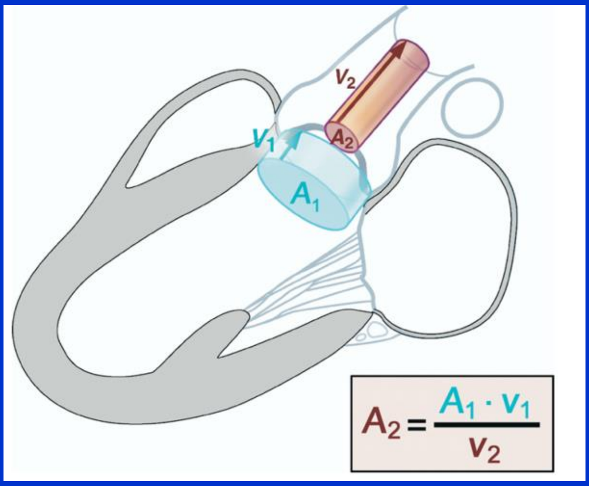
Question

- Evaluation of a patient in the ICU reveals the following:
  - HR 80/min
  - LVOT diameter 2.0 cm
  - LVOT VTI 15 cm
  - Mitral VTI 12 cm
  - BP 130/70 mm Hg
- What is the cardiac output?
  - A. 3.0 L/min
  - B. 3.3 L/min
  - C. 3.8 L/min
  - D. 4.0 L/min
  - E. 4.5 l/min

Doppler Hemodynamic Calculations

- Continuity Principle
  - Conservation of mass
    - What mass flows in must flow out
  - Used in calculations of stenotic orifice areas and regurgitant orifice areas and regurgitant volumes and fractions
- **BASIC EQUATION**
  - $Q_1 = Q_2$
    - $CSA_1 \times VTI_1 = CSA_2 \times VTI_2$

Continuity Principle



Baumgartner H et al. J Am Soc Echocardiogr 2009

Question

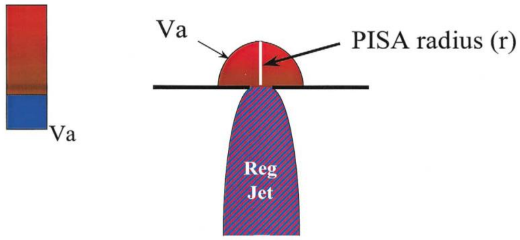
- A 70 year-old man is being evaluated for aortic stenosis. The following measurements were obtained.
  - LVOT diameter 2.0 cm
  - AV peak gradient 48 mm Hg
  - Aortic valve mean gradient 32 mm Hg
  - Aortic valve VTI 63 cm
  - LVOT VTI 20 cm
- The AVA is calculated to be which of the following?
  - A. 0.6 cm^2
  - B 0.8 cm^2
  - C. 1.0 cm^2
  - D. 1.2 cm^2
  - E. 1.4 cm^2

Continuity Principle

- Proximal Isovelocity Surface Area (PISA)
  - Estimates volumetric flow based on color flow Doppler and spectral Doppler measurements
  - As flow approaches a restrictive orifice, isovelocity shells of increasing velocity and reducing area (radius) are formed.
- Used most commonly in calculations of regurgitant orifice area (EROA), regurgitant volumes, and regurgitant fractions.
  - Can be used to calculate stenotic orifice areas

PISA—Basic Equation

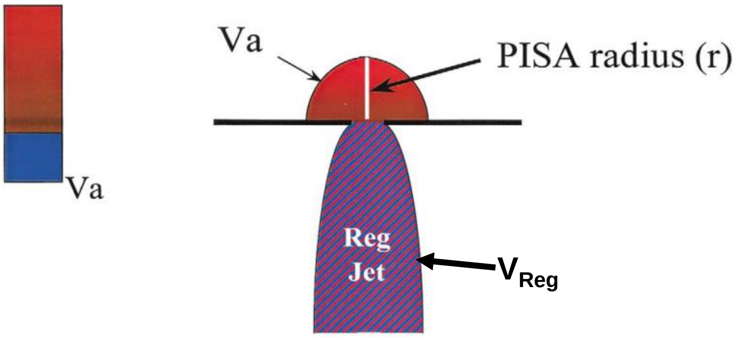
Flow Convergence Method



$$Q = 2\pi r^2 \text{ x } V_a$$

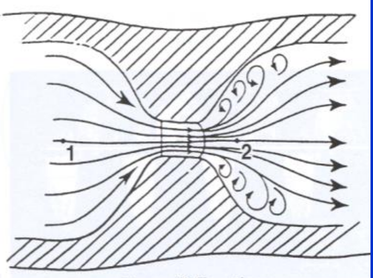
Zoghbi WA et al. J Am Soc Echocardiogr 2003;16:777.

Flow Convergence Method



$$\text{Reg Flow} = 2\pi\ r^2 \text{ x Va}$$
$$\text{EROA} = \text{Reg Flow} / \text{PkV}_{\text{Reg}}$$

Bernoulli Equation



Bernoulli Equation

$$P_1 - P_2 = \underbrace{\frac{1}{2}\rho(V_2^2 - V_1^2)}_{\text{Convective acceleration}} + \underbrace{\rho\int_1^2 \frac{d\bar{v}}{dt}\,\vec{ds}}_{\text{Flow acceleration}} + \underbrace{R(\vec{V})}_{\text{Viscous friction}}$$

P_1 = pressure at location 1
P_2 = pressure at location 2
ρ = mass density of the blood 1.06×10^3 kg/m^3
V_1 = velocity at location 1
V_2 = velocity at location 2

Generally can ignore flow acceleration and viscous friction terms

Modified Bernoulli Equation
$\Delta P = 4V^2$

If V1 >1.5 m/sec it must be included!!
$\Delta P = 4\,(V_2^2 – V_1^2)$

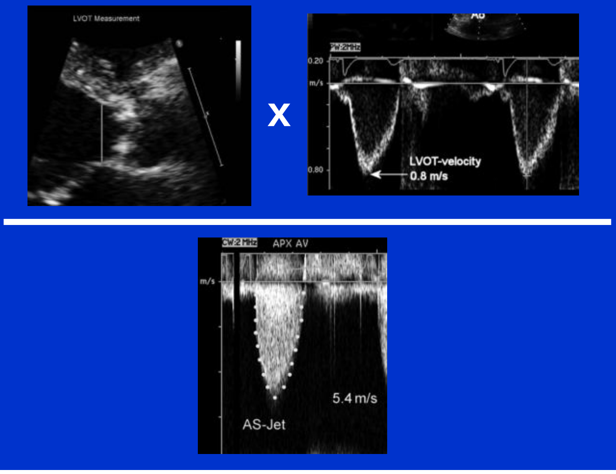
Valve Area Calculations— Stenotic Valve Lesions

Valve Area Calculations

- Continuity Equation
  - $CSA_{prox} \times VTI_{prox} = CSA_{sten} \times VTI_{sten}$
  - $\mathbf{CSA_{sten} = (CSA_{prox} \times VTI_{prox})/VTI_{sten}}$

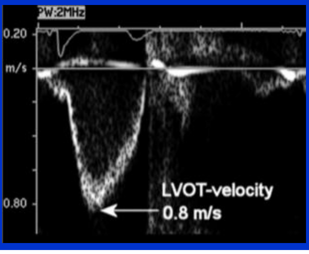
Aortic Valve Area

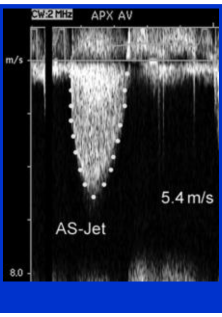
- **AVA = $(CSA_{LVOT} \times VTI_{LVOT})/VTI_{AV}$**



Dimensionless Velocity Ratio

DVR =





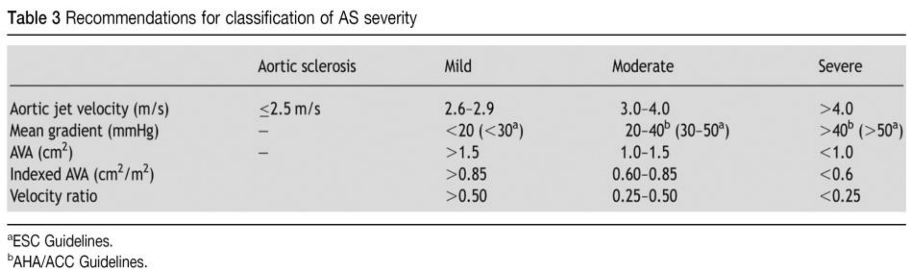


Table 3 Recommendations for classification of AS severity

	Aortic sclerosis	Mild	Moderate	Severe
Aortic jet velocity (m/s)	$\leq$2.5 m/s	2.6–2.9	3.0–4.0	>4.0
Mean gradient (mmHg)	–	<20 (<30[a])	20–40[b] (30–50[a])	>40[b] (>50[a])
AVA (cm^2)	–	>1.5	1.0–1.5	<1.0
Indexed AVA (cm^2/m^2)		>0.85	0.60–0.85	<0.6
Velocity ratio		>0.50	0.25–0.50	<0.25

[a] ESC Guidelines.
[b] AHA/ACC Guidelines.

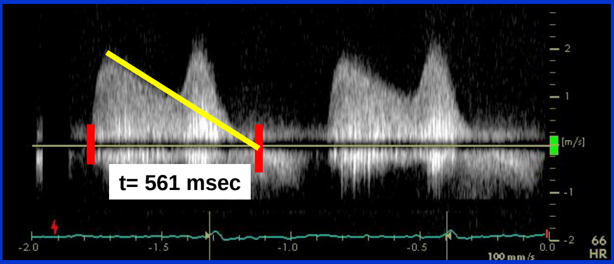
Baumgartner H et al. J Am Soc Echocardiogr 2009

Mitral Valve Area

- Pressure half-time
  - **MVA = 220/PHT**
- Deceleration time
  - **PHT = 0.29 x DT**
  - **MVA = 759/DT**

Question

The best estimate of the mitral orifice area is:

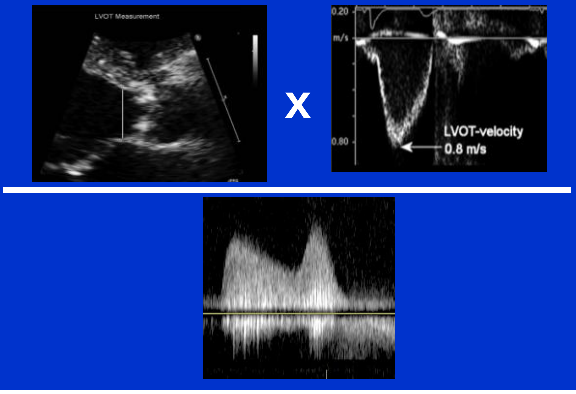


- A. 0.40 cm^2
- B. 0.75 cm^2
- C. 1.0 cm^2
- D. 1.4 cm^2
- E. 2.6 cm^2

Mitral Valve Area

- Continuity Equation
  - **$MVA = (CSA_{LVOT} \times VTI_{LVOT})/VTI_{MV}$**

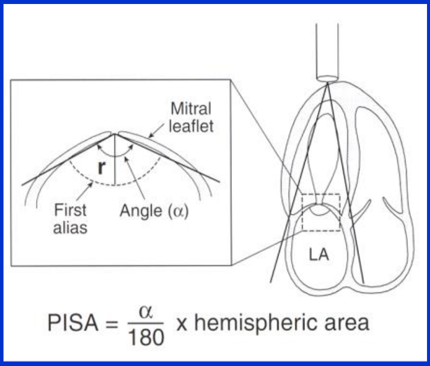
MVA =



Mitral Valve Area—PISA

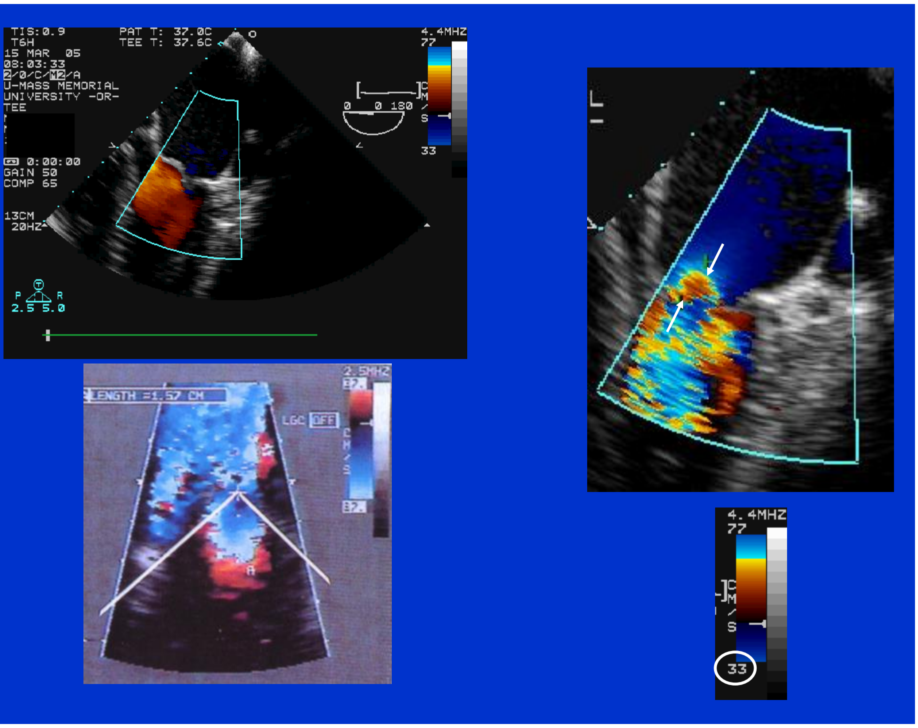
By continuity, boundary flow equals flow across the stenotic mitral orifice (MVA x V_p)

- Thus: $2\pi R^2 \times V_n = MVA \times V_p$
- Rearranging:
  - $MVA = 2\pi R^2 \times V_n / V_p$
- Due to doming, angle correction must be used



$$PISA = \frac{\alpha}{180} \times \text{hemispheric area}$$

$$\mathbf{MVA = \frac{(2\pi R^2 \times Vn)}{Vp \times (a/180)}}$$



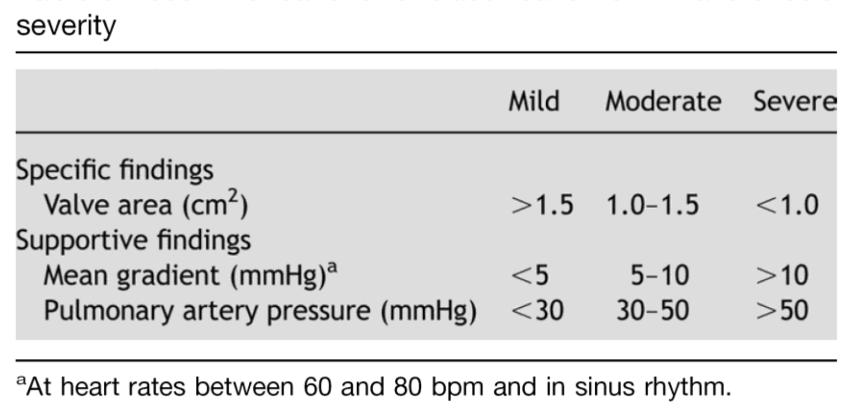


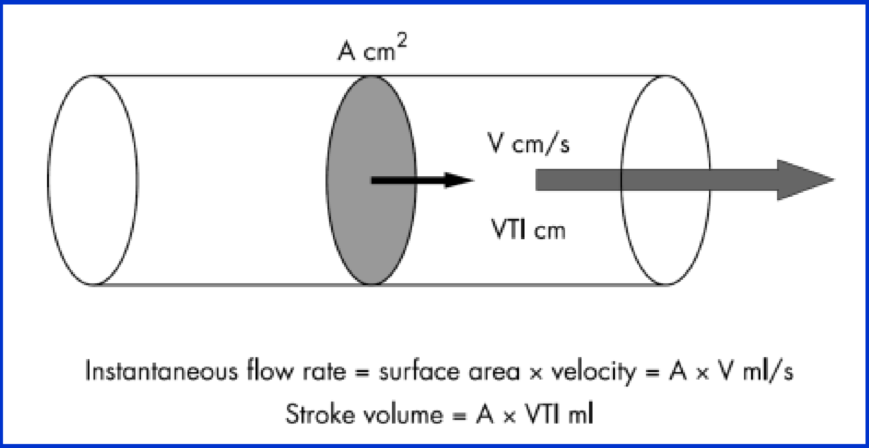
Table 9 Recommendations for classification of mitral stenosis severity

	Mild	Moderate	Severe
Specific findings			
Valve area (cm^2)	>1.5	1.0–1.5	<1.0
Supportive findings			
Mean gradient (mmHg)[a]	<5	5–10	>10
Pulmonary artery pressure (mmHg)	<30	30–50	>50

[a] At heart rates between 60 and 80 bpm and in sinus rhythm.

Baumgartner H et al. J Am Soc Echocardiogr 2009

Quantification of Regurgitant Lesions

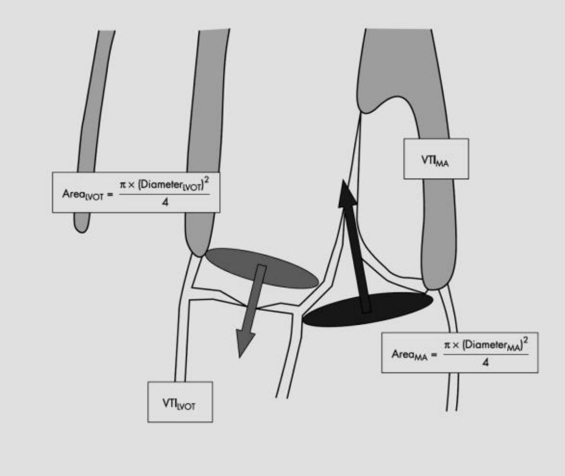


Instantaneous flow rate = surface area × velocity = A × V ml/s

Stroke volume = A × VTI ml

Irvine T et al. Heart 2002;88(Suppl IV):iv11

Continuity



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Mitral Regurgitation
Pulsed Doppler Quantitation

- Stroke volume = CSA x VTI
  - **$\pi D^2/4$ x VTI = 0.785 D^2 x VTI**
  - Regurgitant volume (RVol)
    - RVol = SV_{RV} - SV_{CV}
    - RVol = MV inflow - AV outflow

$(CSA_{MV} \times VTI_{MV}) - (CSA_{LVOT} \times VTI_{LVOT})$

  - Regurgitant fraction (RF)

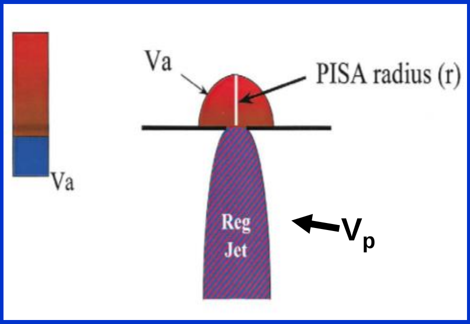
RF = (RVol / MV flow) x 100

  - Effective Regurgitant Orifice Area (ERO)

ERO = RVol/VTI_{MR}

Proximal Isovelocity Surface Area (PISA) Method

By continuity,
boundary flow
($2\pi R^2 \times V_a$) equals
flow across the
regurgitant orifice
(ERO x V_p)

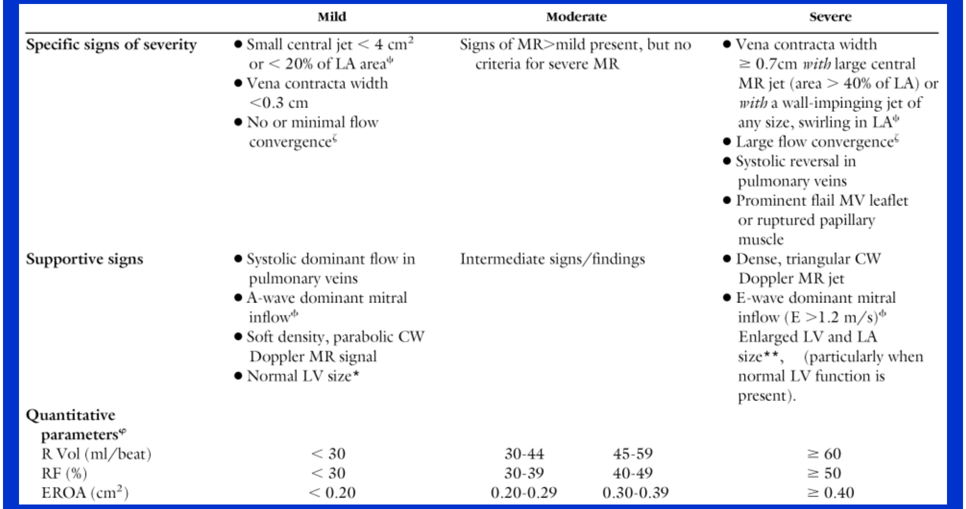


$$EROA = 2\pi R^2 \times V_a / V_p$$

Let’s Do the Math!

- $ERO_{MR} = 2\pi R^2 \times (V_N / V_P)$
  - $ERO_{MR} = 6.28\ (0.54 \times 0.54) \times (32/439)$
  - $ERO_{MR} = 0.13\ cm^2$
- Simplified method
  - Set Nyquist limit at 40 cm/s and assume that MR velocity is 500 cm/s
  - Then: $ERO = R^2 / 2$

Summary: MR Severity



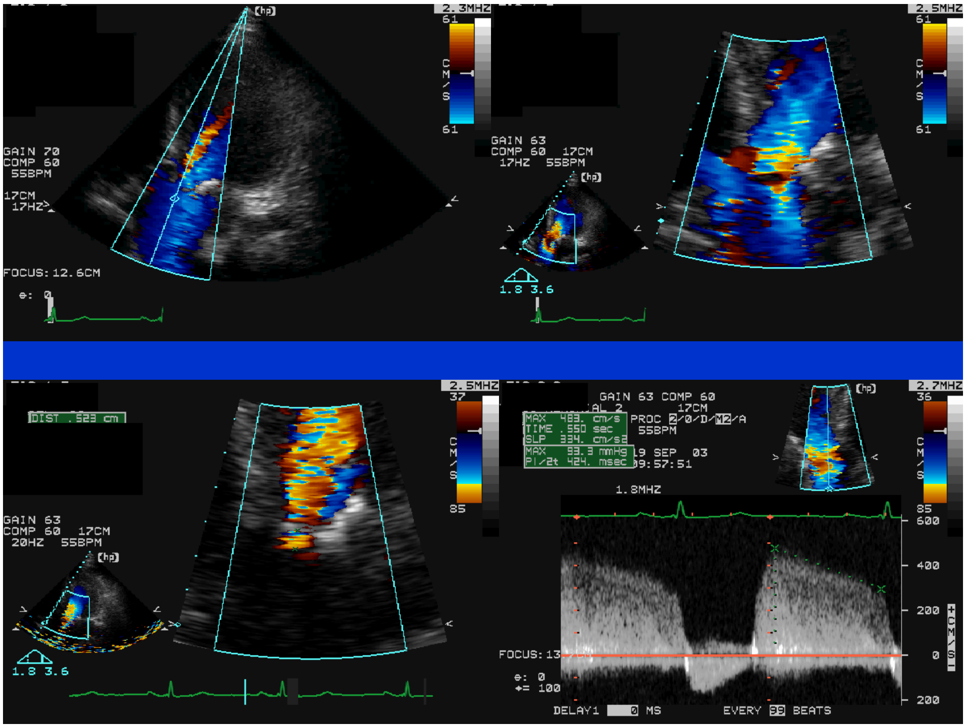
	Mild	Moderate		Severe
Specific signs of severity	● Small central jet $< 4\ cm^2$ or $<$ 20% of LA area[ψ] ● Vena contracta width $<$0.3 cm ● No or minimal flow convergence[ζ]	Signs of MR>mild present, but no criteria for severe MR		● Vena contracta width $\geq$ 0.7cm *with* large central MR jet (area $>$ 40% of LA) or *with* a wall-impinging jet of any size, swirling in LA[ψ] ● Large flow convergence[ζ] ● Systolic reversal in pulmonary veins ● Prominent flail MV leaflet or ruptured papillary muscle
Supportive signs	● Systolic dominant flow in pulmonary veins ● A-wave dominant mitral inflow[Φ] ● Soft density, parabolic CW Doppler MR signal ● Normal LV size*	Intermediate signs/findings		● Dense, triangular CW Doppler MR jet ● E-wave dominant mitral inflow (E >1.2 m/s)[Φ] Enlarged LV and LA size**, (particularly when normal LV function is present).
Quantitative parameters[φ]				
R Vol (ml/beat)	< 30	30-44	45-59	≥ 60
RF (%)	< 30	30-39	40-49	≥ 50
EROA (cm^2)	< 0.20	0.20-0.29	0.30-0.39	≥ 0.40

Aortic Regurgitation

- Quantitative Doppler
  - Regurgitant volume (RVol)
    - RVol = AV flow - MV flow
      - AV flow = LVOT annulus area x AV VTI
      - MV flow = MV annulus area x MV VTI
  - Regurgitant fraction (RF)
    - RF = (RVol / AV flow) x 100

Aortic Regurgitation: Quantification

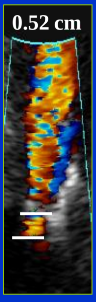
- Proximal Isovelocity Surface Area method
  - Continuity principle
  - Effective regurgitant orifice (ERO) area
    - EROA = flow rate/peak AR velocity
    - **EROA = $(2 \pi R^2 \times V_{alias}) / V_{peak}$**



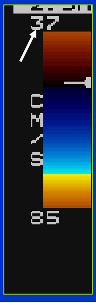
Let’s Do the Math!

- **$ERO_{AR} = 2\pi R^2 \times (V_{alias} / V_{peak})$**
  - $ERO_{AR} = 6.28\ (0.52 \times 0.52) \times (37/483)$
  - $ERO_{AR} = 0.13\ cm^2$

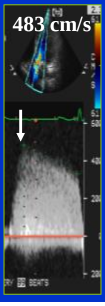
R



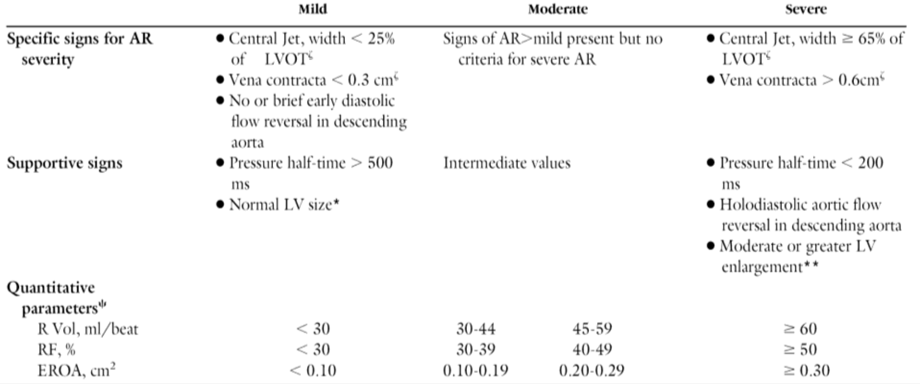
V_{alias}



V_{peak}

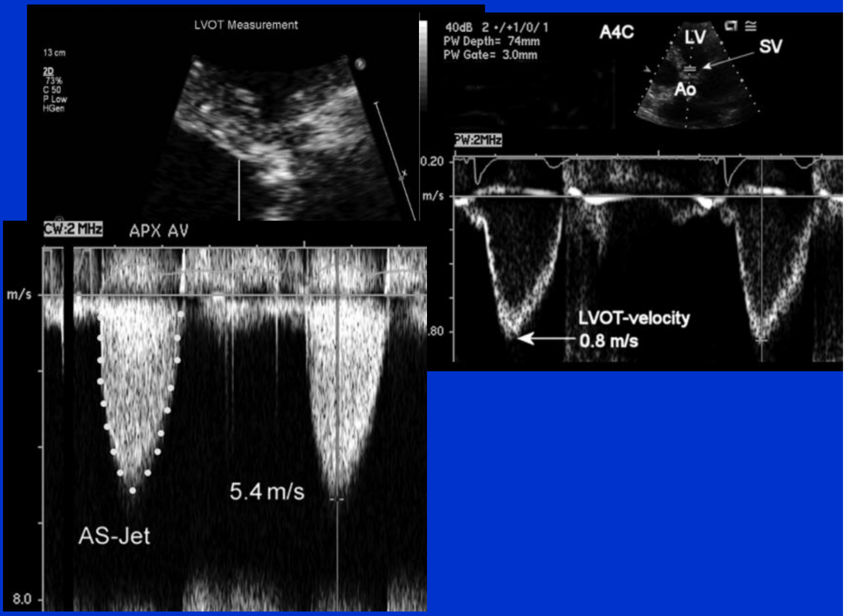


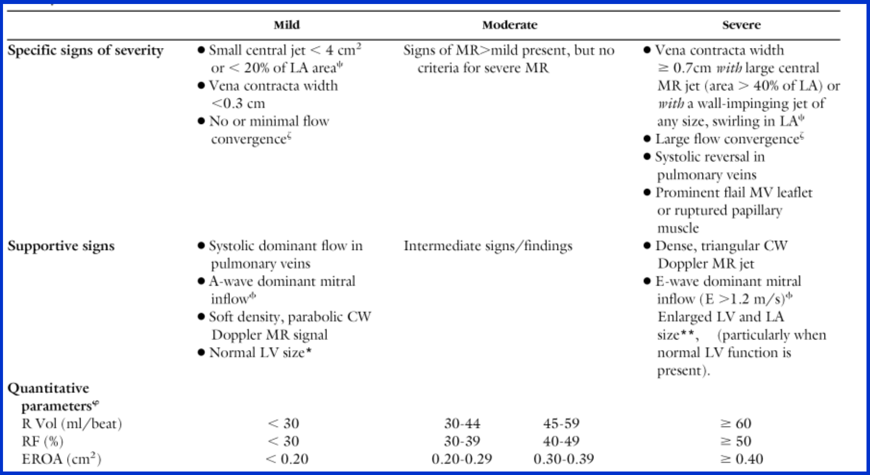
Summary: AR Severity



	Mild	Moderate		Severe
Specific signs for AR severity	• Central Jet, width < 25% of LVOT$^{\zeta}$ • Vena contracta < 0.3 cm$^{\zeta}$ • No or brief early diastolic flow reversal in descending aorta	Signs of AR>mild present but no criteria for severe AR		• Central Jet, width ≥ 65% of LVOT$^{\zeta}$ • Vena contracta > 0.6cm$^{\zeta}$
Supportive signs	• Pressure half-time > 500 ms • Normal LV size*	Intermediate values		• Pressure half-time < 200 ms • Holodiastolic aortic flow reversal in descending aorta • Moderate or greater LV enlargement**
Quantitative parameters$^{\psi}$				
R Vol, ml/beat	< 30	30-44	45-59	≥ 60
RF, %	< 30	30-39	40-49	≥ 50
EROA, cm^2	< 0.10	0.10-0.19	0.20-0.29	≥ 0.30

Ventricular Function





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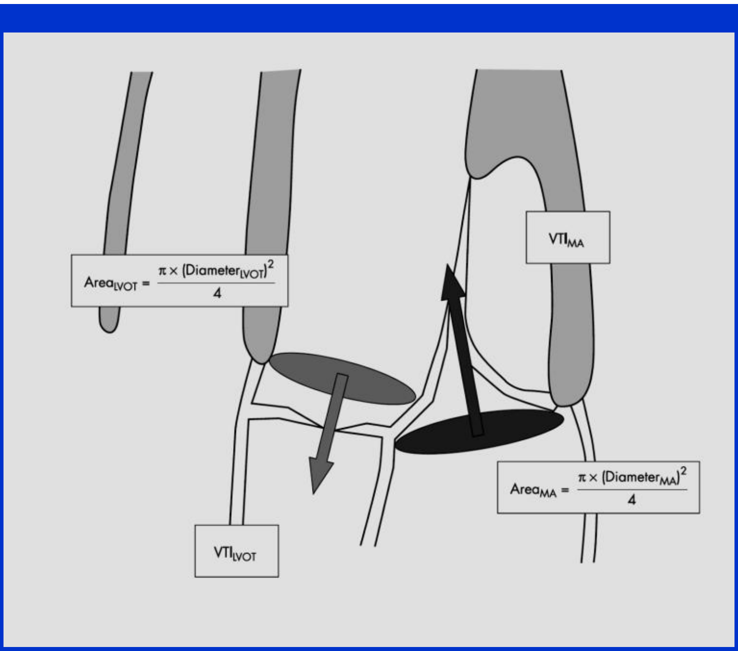
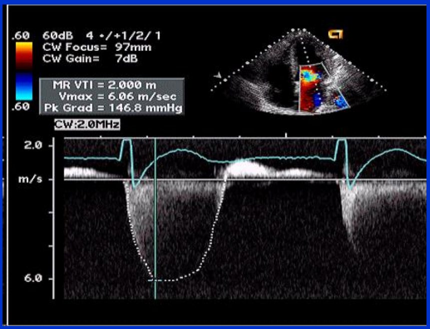


Irvine T et al. Heart 2002;88(Suppl IV):iv11

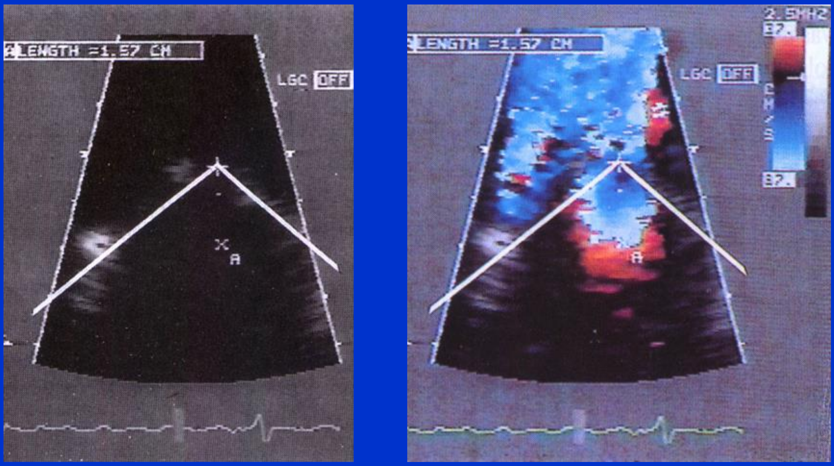
How to do PISA-III

- Measure the peak velocity of the MR jet as parallel to flow as possible
- Trace the spectral profile to determine the VTI of the MR jet



MR VTI = 2.000 m
Vmax = 6.06 m/sec
Pk Grad = 146.8 mmHg

Mitral Stenosis



Rifkin RD et al. J Am Coll Cardiol 1995;26:458.